Optimized wavelet preconditioning

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Abstract The numerical solution of linear stationary variational problems involving elliptic partial differential operators usually requires iterative solvers on account of their problem size. Our guiding principle is to devise theoretically and practically efficient iterative solution schemes which are optimal in the number of arithmetic operations, i.e., of linear complexity in the total number of unknowns. For these algorithms, asymptotically optimal preconditioners are indispensable. This article collects the main ingredients for multilevel preconditioners based on wavelets for certain systems of elliptic PDEs with smooth solutions. Specifically, we consider problems from optimal control with distributed or Dirichlet boundary control constrained by elliptic PDEs. Moreover, the wavelet characterization of function space norms will also be used in modelling the control functional, thereby extending the range of applicability over conventional methods. The wavelet preconditioners are optimized for these PDE systems to exhibit small absolute condition numbers and consequently entail absolute low iteration numbers, as numerical experiments show.

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