An Optimization-Based Empirical Mode Decomposition (OEMD)

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Traditional Empirical Mode Decomposition (EMD)

Huang *et al.* [1]: Empirical Mode Decomposition (EMD) decomposes signal f additively into intrinsic mode functions (IMFs) $f_k(t)$

$$f(t)=\sum_{k=1}^{n} f_k(t)+r_{n+1}(t) \quad (1)$$

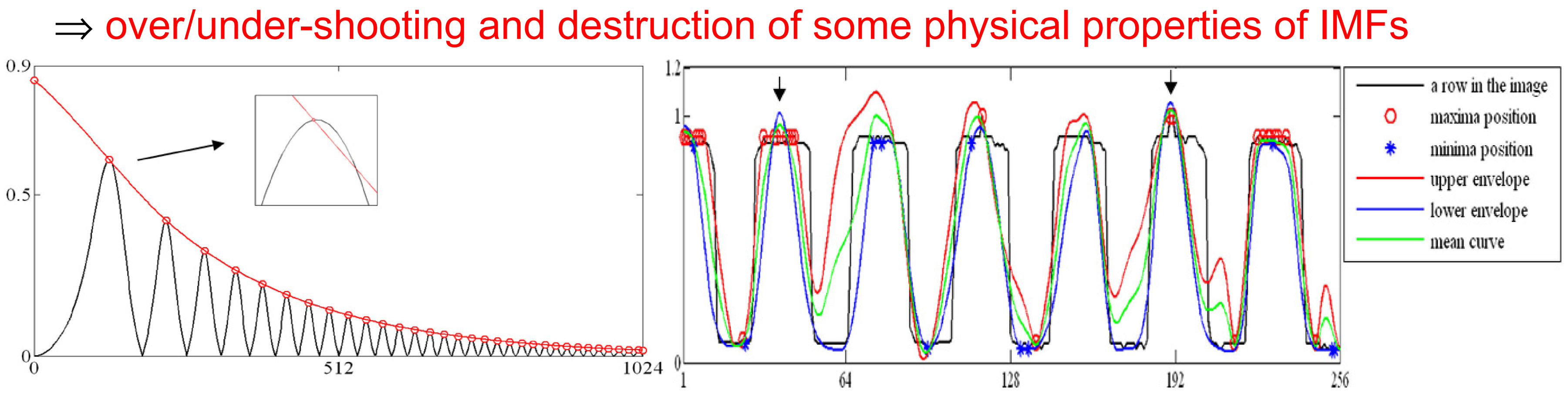
For k-th IMF: $a_k(t) > 0$ is envelope

Definition ⇒? $a_k(t) \geq f(t) \geq -a_k(t)$ for all t (2)

Equality holds at extrema $f(t_i^+)$ or $f(t_i^-)$

Generation of envelope by cubic spline interpolation of extrema

⇒ over/under-shooting and destruction of some physical properties of IMFs

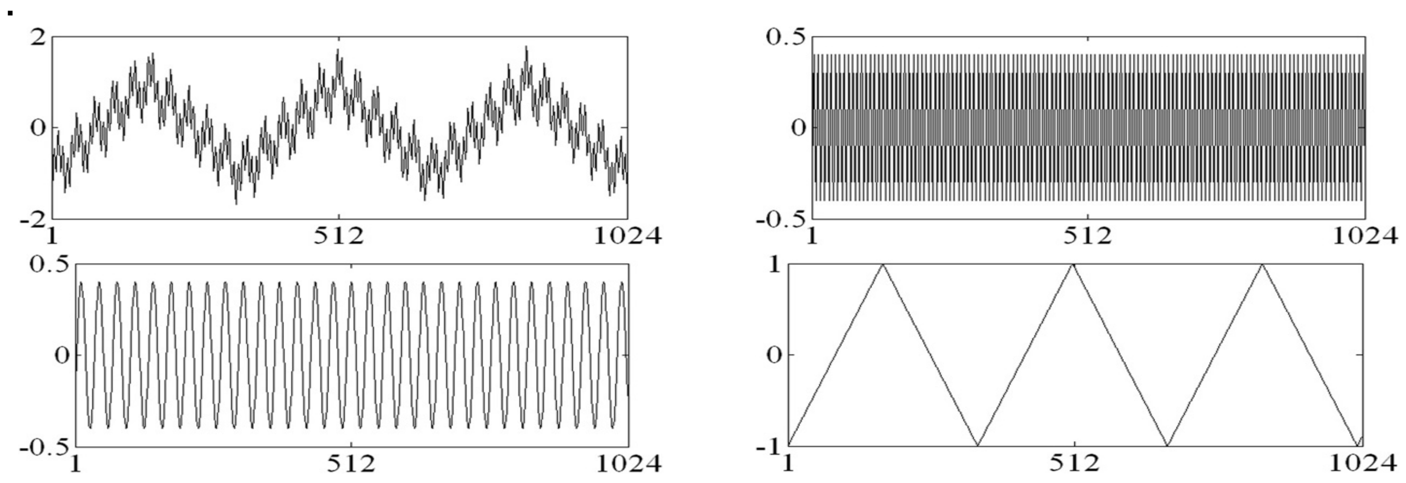


Hou *et al.* [2]: EMD $\quad f(t)=\sum_{k=1}^{n} a_k(t)\cos(\theta_k(t))+r_{n+1}(t)$ (1a)

1-D OEMD

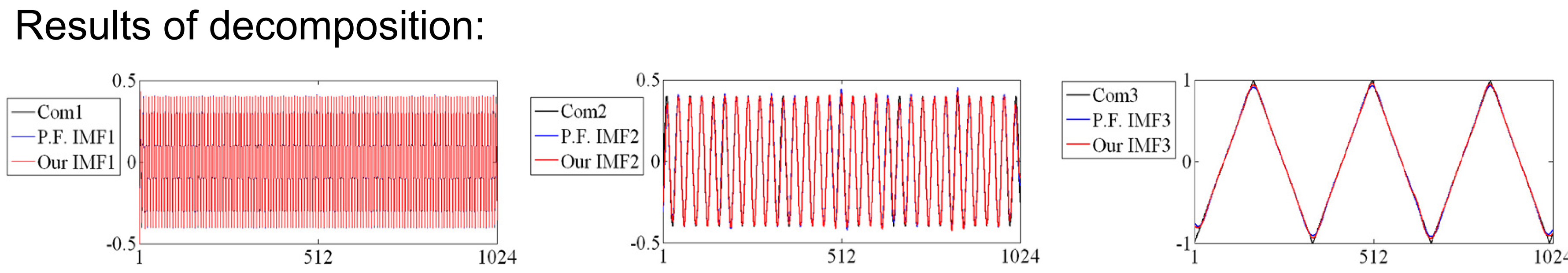
Smoothness functional $\boldsymbol{S}(\cdot)$: 3 rd order total variation

Test signal from [5] consisting of three components: a sinusoid of some medium period T superimposed by two triangular waveforms with periods smaller than larger than T.

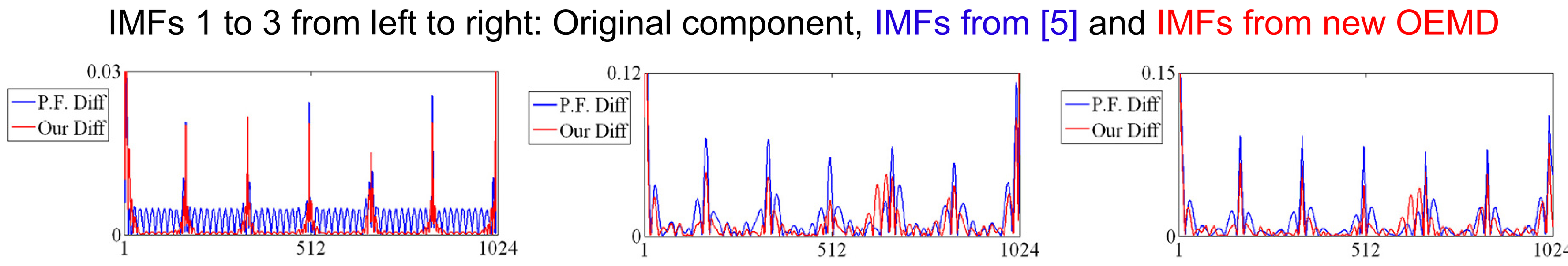


Original signal from [5]

Results of decomposition:



IMFs 1 to 3 from left to right: Original component, IMFs from [5] and IMFs from new OEMD



Difference of original components and IMFs: difference between original components and [5], difference between original components and OEMD

Instantaneous Frequency Analyses

Improvement of normalization based instantaneous frequency analyses from [6]:

Step 1: solve optimization problem

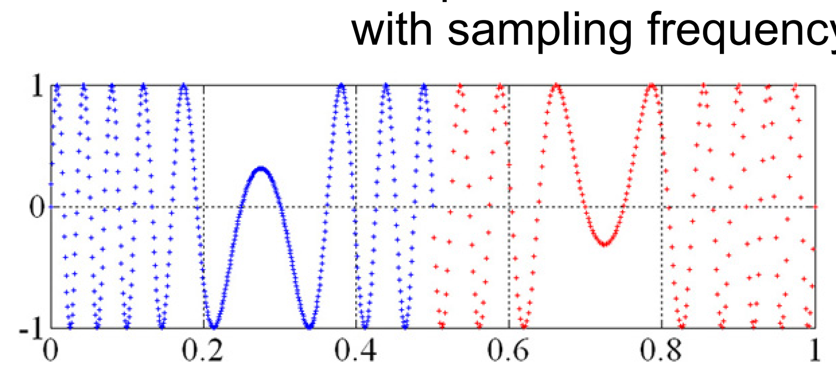
(P3) Minimize $\|\boldsymbol{S}(X(t))\|_2$ over all $X(t)$

Subject to $|f_k(t)| \leq X(t)$, $f_k(t_i^+) = X(t_i^+)$ and $|f_k(t_i^-)| = X(t_i^-)$ (4)

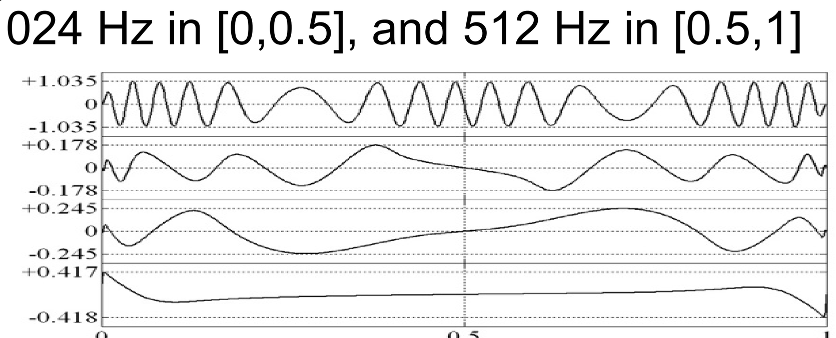
Solution: $e_k(t)$ is envelope of $|f_k(t)|$

Step 2: define $\cos(\theta_k(t)) := \dfrac{f_k(t)}{e_k(t)}$

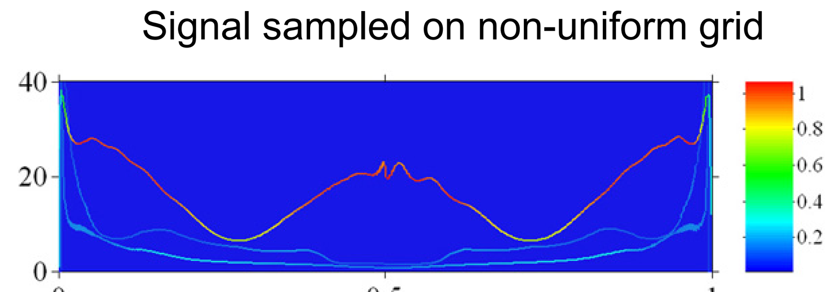
Example from [7]: signal $f(t) = \sin(8\pi(\sin(2\pi t) + t))$, $t \in [0,1]$ sampled on non-uniform grid with sampling frequency 1024 Hz in [0,0.5], and 512 Hz in [0.5,1]



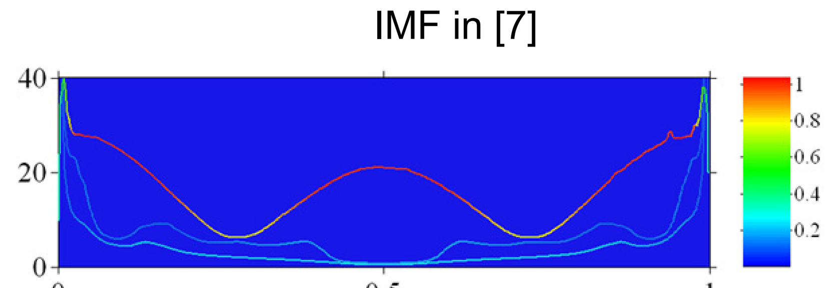
Signal sampled on non-uniform grid



IMF in [7]



Amplitude spectrum in [7]



Our amplitude spectrum

Optimization-Based Empirical Mode Decomposition

New Optimization-Based Empirical Mode Decomposition (OEMD) defines envelope for (1) strictly mathematically through optimization process:

(P1) Minimize $\|\boldsymbol{S}(X(t))\|_2$ over all $X(t)$

Subject to $f_k(t) \leq X(t)$, and $f_k(t_i^+) = X(t_i^+)$

(P2) Minimize $\|\boldsymbol{S}(X(t))\|_2$ over all $X(t)$ (3)

Subject to $f_k(t) \geq X(t)$, and $f_k(t_i^-) = X(t_i^-)$

Here: $f_k(t)$ given function in k-th sifting process

t_i^+, t_i^- position of maxima, minima

$f_k(t_i^+)$, $f_k(t_i^-)$ local maxima, minima

$\boldsymbol{S}(\cdot)$ smoothness functional (e.g, n-th derivative, n-th total variation)

Solutions: $a_k^+(t)$, $a_k^-(t)$ upper, lower envelope

(See [2] for a different optimization approach)

Solution of optimization problems (3) by CVX tool box [3,4].

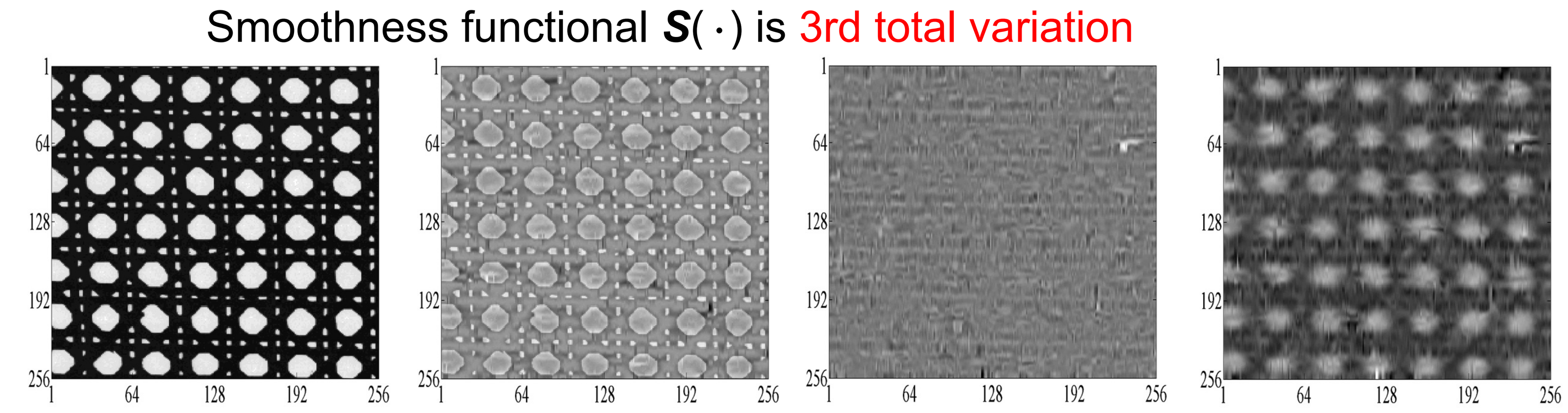
2-D OEMD

Naive 2-D OEMD:

compute extrema in each direction independently in tensor product approach (compare to 2-D wavelet decomposition in JPEG-2000)

Test image: A portion of D102 in the Brodatz image set [8]

Smoothness functional $\boldsymbol{S}(\cdot)$ is 3rd total variation



(a) Test image (b) IMF1 (c) IMF2 (d) Residual image

Observation: many noise-like stripes in every IMF image

Reason:

method does not consider local extrema of image in both directions simultaneously

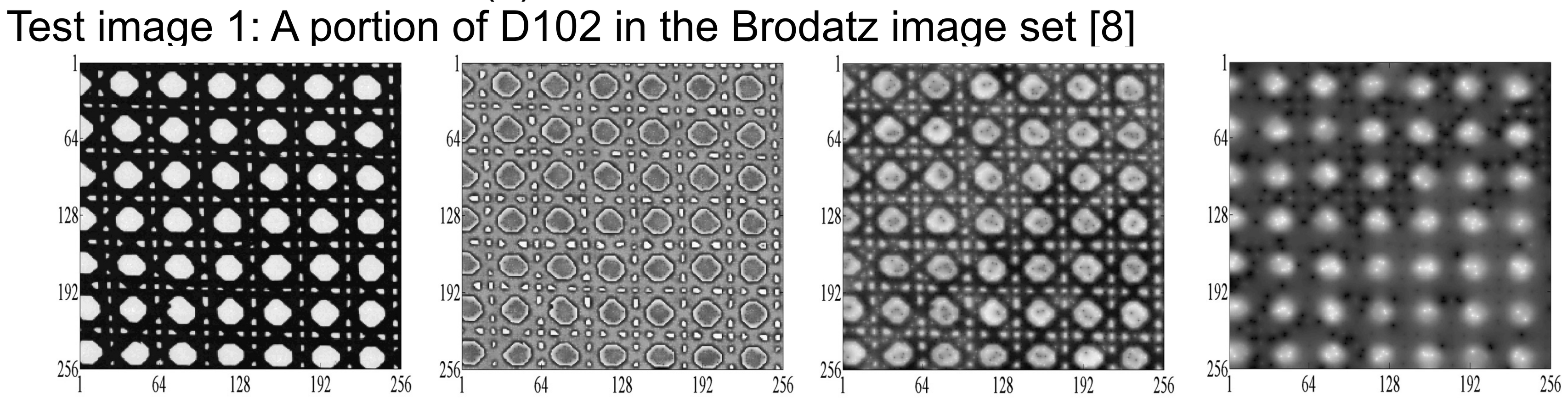
Bivariate OEMD:

Step 1: find extrema of image by comparing pixel value with all neighbors in $k \times k$ subimage

Step 2: solve optimization problems (P1) and (P2) for $X(i, j)$

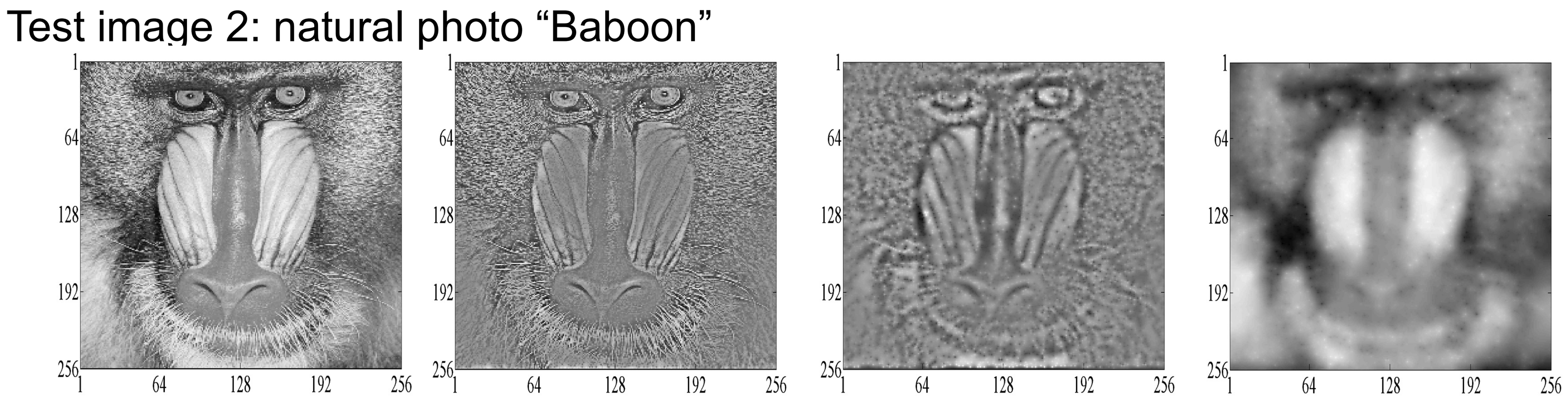
Smoothness functional $\boldsymbol{S}(\cdot)$ is first order 2-D total variation

Test image 1: A portion of D102 in the Brodatz image set [8]



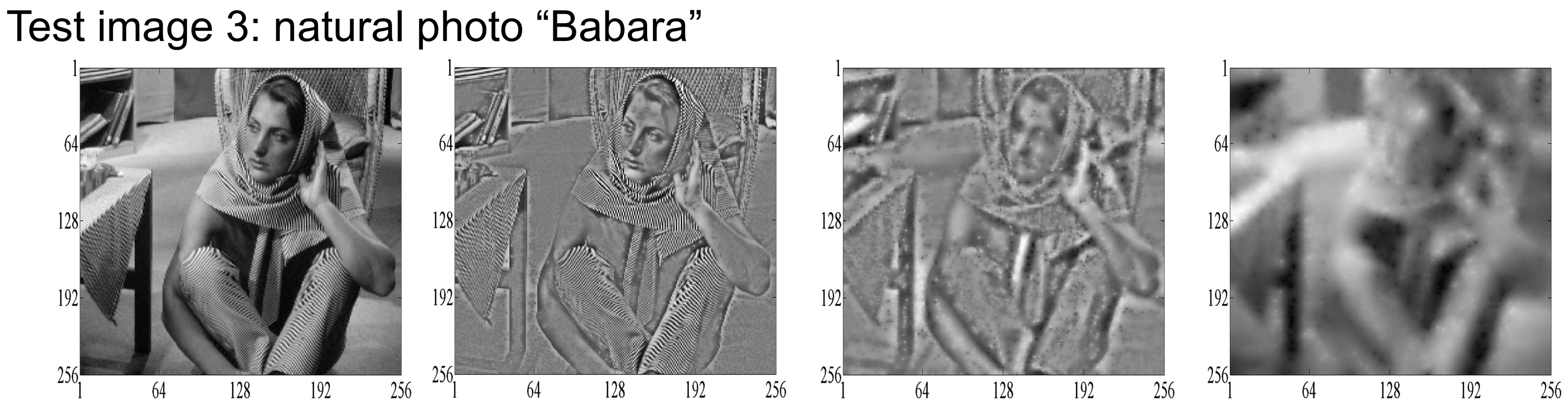
(a) Test image (b) IMF1 (c) IMF2 (d) Residual image

Test image 2: natural photo "Baboon"



(a) Test image (b) IMF1 (c) IMF2 (d) Residual image

Test image 3: natural photo "Babara"



(a) Test image (b) IMF1 (c) IMF2 (d) Residual image

References:

[1] Huang, N., Shen, Z., Long, S., Wu, M., Shih H., Zhang, Q., Yen, N., Tung, C. and Liu, H. (1998): The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. Proceedings of the Royal Society of London A 454, 903-995.
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